B FERROMAGNETIC ACTUATORS

B.1 Linear Actuator - Introduction

The basic concept for the exploitation of the forces that tend to minimise the reluctance in electromagnetic circuits can be demonstrated using a ferromagnetic actuator, Figure B-1. This consists of a core and a moveable armature (horizontal movement in figure). On to the core is wound a coil with N turns, while the armature is connected to a mechanical load such as a fixed load or a spring. A magnetic flux will be produced in the air gap between the core and the armature when a current, I, flows in the coil.

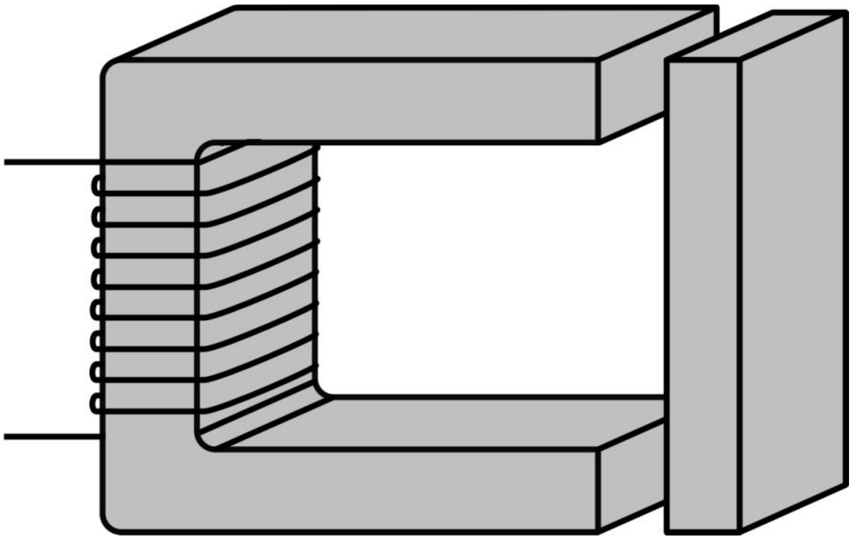


Figure B-1: Ferromagnetic actuator

B.1.1 *Actuator Characterisation – Part 1: Inductance*

The magnetic flux in the circuit can be found by considering the magneto-motive force, *mmf*, magnetic field intensity, H, and the flux path, l. (Amperes law):

(Eq B-1)

Ignoring leakage flux and assuming ferromagnetic material is not saturated the mmf in terms of air-gap flux density, B_a, and gap length is:

(Eq B-2)

Magnet flux is conserved, thus:

(Eq B-3)

where and A_{eff} is the effective gap area taking into account magnetic field fringing. Note that field fringing tends to increase as the air-gap is increased.

B.1.1.1 *Calculation of Effective Air-Gap Width*

Two simplified cases of effective areas of air-gaps are presented here. Precise calculation of the air-gap would need to be able to determine exact patterns of fringing (possible with FEA software).

Equal Areas

When the areas of the two parts of the ferromagnetic core are equal, as is Figure B-2:

Figure B-2: Effective air gap width calculation - Equal Areas

(Eq B-4)

Different Areas

When the areas of the ferromagnetic core on one side of the air-gap is larger than the other, such as on a corner, shown in Figure B-3:

Figure B-3: Effective air gap width calculation - Different Areas

(Eq B-5)

Where W and T represent the width and depth of the part with the smallest area.

Finally we have the permeability of each of the regions:

(Eq B-6)

Combining the above gives the expression:

(Eq B-7)

Which can be re-arranged to give:

(Eq B-8)

Figure B-4: Equivalent magnetic circuit

The inductance is then found:

(Eq B-9)

Which can be expressed in terms of the air-gap:

(Eq B-10)

The variation in inductance with gap thus follows the relationship:

(Eq B-11)

where:

(Eq B-12)

B.1.2 *Energy and Work in Simply Excited Actuators – Part 1: Analytical*

B.1.2.1 *Flux linkage and emf*

With the movable armature, the inductance is variable and decreases as the air gap is increased. The emf of the actuator is the rate of change of flux linkage, Ψ, which is now a function, not only of the inductance of the winding, but of the rate of change of inductance of the winding.

(Eq B-13)

where x is the air-gap, and it is assumed that L is not a function of current and other non-linear effects. This expression is separated into components due to both armature movement and current fluctuation giving:

(Eq B-14)

B.1.2.2 *Energy balance.*

The energy relationships of the system are

= + +

The energy losses are electrical loss due to the winding resistance, field loss due to hysteresis and eddy currents, and mechanical loss due to friction and windage.

Ignoring the losses, the motion of the armature through a positive distance, dx, in a time, dt, will produce a change in the relationship between the electrical energy (W_e), the mechanical energy (W_m) and the stored energy in the field (W_f), such that

(Eq B-15)

In practice, we observe that

-
-

Using the energy balance described above we will now explain the two phenomena (i.e. determine how energy is distributed in the actuator based on a slow/fast displacement).

B.1.2.3 *Electrical Energy*

Energy in a circuit is the integral of power over time, hence:

(Eq B-16)

Substituting for the e.m.f. due to the inductance as given in **(Eq B-14)**

(Eq B-17)

B.1.2.4 *Mechanical Energy.*

The mechanical work done is related to the force exerted in the x-direction in order to move the load, F_x, a distance dx by:

(Eq B-18)

and hence the force exerted:

(Eq B-19)

The force is always in the direction that leads to an increase in inductance (decrease in reluctance), and hence tends to close the air gap.

B.1.2.5 *Stored energy in coil.*

If the relative permeability of the core material is regarded as constant or the permeability is infinite, then the energy supplied to the field over time dt by the supply is

(Eq B-20)

Therefore, the energy stored in the magnetic field in terms of the magnetic flux linkage, Ψ, is:

(Eq B-21)

We know that, from the definition of inductance:

(Eq B-22)

Hence

(Eq B-23)

Therefore, from **(Eq B-21)** and **(Eq B-23)**:

(Eq B-24)

Now if we find the derivative of this stored energy with respect to position:

(Eq B-25)

B.1.2.6 *Actuator Force as a Function of Current and Inductance*

Finally, if we combine these components using the energy balance determined in **(Eq B-15)**, we get;

(Eq B-26)

Which, if we re-arrange, gives:

(Eq B-27)

B.1.3 *Actuator Characterisation – Part 2: Actuator Force and Magnetic Pressure*

If we refer back to our analysis of the actuator shown in Figure B-1 in which effective area of the air gap is the area of the core, A_{eff} and the air-gap length is a variable distance, x, the inductance of the circuit can be written as **(Eq B-11)**:

(Eq B-28)

Remebering that the parameter b was calculated as:

(Eq B-29)

Usually the permeability of the ferromagnetic sections are large, hence $b \rightarrow 0$, then:

(Eq B-30)

Referring back to the evaluation of flux in the core but assuming that the ferromagnetic parts can be neglected as the permeability of these parts is large when compared to that of the air gap:

(Eq B-31)

Therefore (using the air gap length as actuator displacement, $x = g$):

(Eq B-32)

Yields:

(Eq B-33)

Which, if we substitute into the force equation yields an interesting result for the actuator:

(Eq B-34)

with $2A_{eff}$ the total effective air gap area of the two poles of the actuator.

The force per unit area of air gap, $\frac{B_g^2}{2\mu_0}$, is called the *magnetic pressure*.

For a typical $B_g = 1.0\mathbf{T}$ the magnetic pressure is equal to 3.98x10^5 N/m^2 or 0.398 MPa. This value can be compared against the capabilities of hydraulic, pneumatic and mechanical driven actuation systems.

B.1.4 *Energy and Work in Simply Excited Actuators – Part 2: Graphical*

Using the analysis given, we can determine the force and energy relationships for an actuator whose inductance variation with actuator displacement/stroke is known and that is independent of I. That is to say that the core material doesn't saturate and that inductance variation is solely down to the reluctance change of the actuator due to the size of the air-gap.

For problems in which saturation needs to be incorporated, a graphical method can be adopted.

Inductance:

(Eq B-35)

Stored Energy in Magnetic Field as a function of flux linkage and current:

(Eq B-36)

We can define a complementary quantity, co-energy, W_C:

(Eq B-37)

And finally, the energy transferred from the electrical system into the magnetic field is:

(Eq B-38)

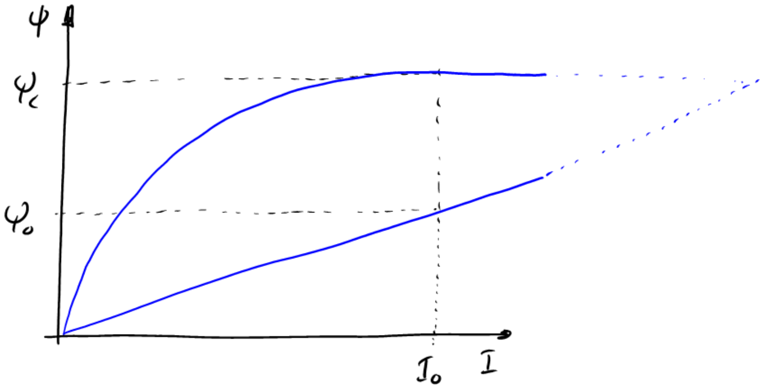
Hence we can draw the following (rather complex) diagram:

Figure B-5: Psi-I diagram with relevant information

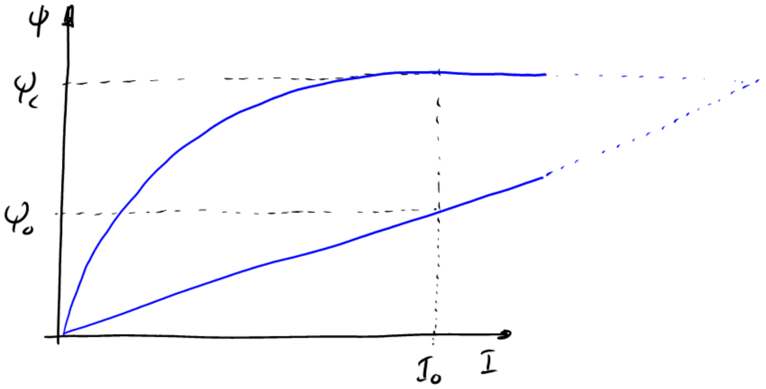
Whilst the basic information contained within a $\Psi - I$ diagram does not change, several slightly different forms exist based on whether the electromechanical device is behaving as a motor or a generator and on the rate at which work is being done.

B.1.4.1 *Work Being Done by External Force on Armature*

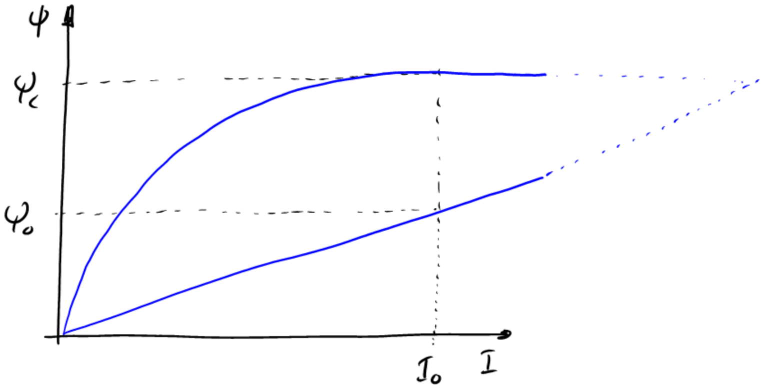
Slow Movement



Fast Movement

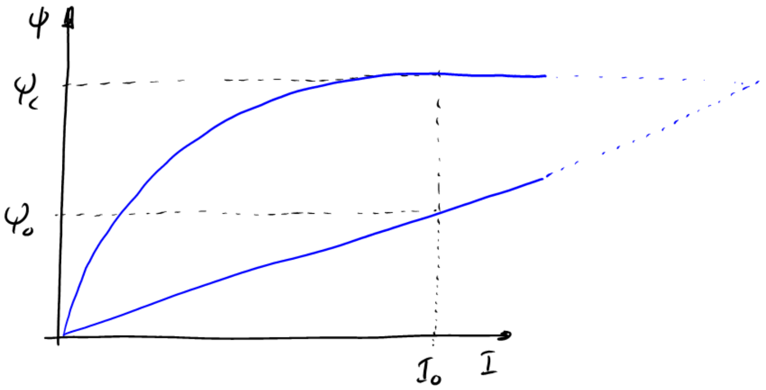


Intermediate Movement

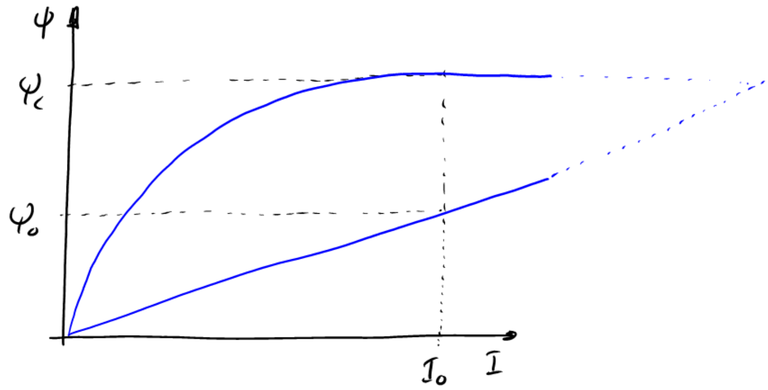


B.1.4.2 *Work Being Done by Magnetic Field on Armature*

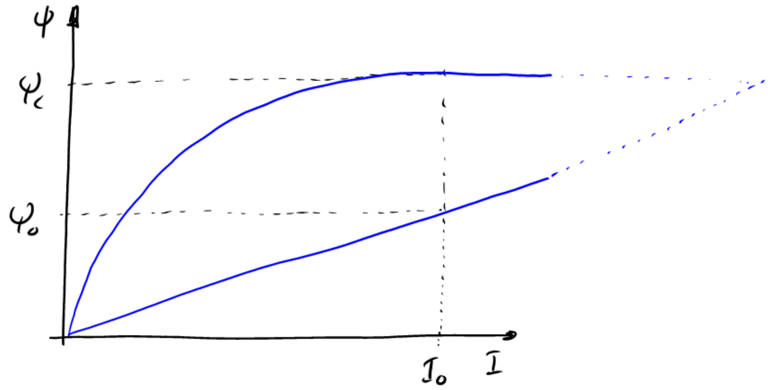
Slow Movement



Fast Movement



Intermediate Movement



B.2 Rotary Actuators

A simple rotary actuator is shown in Figure B-6, together with its flux pattern. Here, unlike the ferromagnetic actuator described above, the minimisation of the reluctance must be by the increase in the air-gap area since the air-gap length is now fixed while the rotor is under the poles of the stator. The flux, Φ, can be written in terms of the *m.m.f.* and the total magnetic circuit reluctance, $\Re_T$:

(Eq B-39)

where $\Re_r$ is the reluctance of the rotor, $\Re_s$ the reluctance of the stator and $\Re_g$ the reluctance of the single air gap given by:

(Eq B-40)

Therefore, assuming the permeability of the magnetic material is large and there is no saturation, the stator and rotor reluctances ($\Re_s$ and $\Re_r$) both tend to zero. The reluctance of the rotary actuator $\Re_T$, is thus dominated by the air gap reluctance:

(Eq B-41)

Figure B-6: Fringing flux on a rotary actuator

The flux will be concentrated in the overlap region between the stator and rotor poles and will tend to produce a radial force whose magnitude depends upon the overlap area and the air-gap length. The radial force will not, however, produce any torque on the rotor since the torque requires a force tangential to the rotor poles. The only force tangential to the rotor is due to the fringing flux at the edges of the rotor and stator.

The effective air gap area is given by:

(Eq B-42)

Where l is the active length **(m)**, r is the rotor radius **(m)**, θ the overlap angle **(radians)** and A_f is the area of the fringing fields **(m^2)**.

Therefore we can calculate the actuator flux:

(Eq B-43)

In order to work out the torque produced by such a machine it is again necessary to consider energy changes. The technique demonstrated for a linear actuator can be applied to a rotating system. For such a system, the torque, T, and overlap angle between the rotor and the stator, θ, are required to calculate the mechanical energy:

(Eq B-44)

Thus, the torque can be expressed in terms of mechanical energy as:

(Eq B-45)

As before, the inductance can be written as:

(Eq B-46)

The second term on the right-hand side assumes that the fringing field is proportional to the m.m.f. produced by the stator coil. The inductance of the circuit is:

(Eq B-47)

Allowing, finally, the torque to be written as:

(Eq B-48)

It can be seen that the torque is independent of the overlap angle and is proportional to the square of the current. This means that the torque is also independent of the direction of the flow of current in the coils and whether alternating or direct current is used. This reluctance force effect is therefore employed in moving iron instruments.

Although the fringing field term does not appear to affect the torque, the expression is only valid over the range of stator-rotor overlap where the fringing flux pattern remains constant.

B.3 Specific Pressures of Various Actuating Systems

B.3.1 *Magneto-static*

In section B.1.3 we found that the magnetic pressure for a ferromagnetic actuator (neglecting losses and flux leakages) could be approximated by:

(Eq B-49)

It is a useful exercise to look at the same parameter for a variety of actuation technologies to allow the correct technology to be used in the correct situation.

B.3.2 *Electro-static*

Consider a capacitor made from a pair of plates separated by a di-electric with permittivity, ε. The force between two plates of a capacitor separated by a distance x is equal to:

(Eq B-50)

And the energy

(Eq B-51)

Hence:

(Eq B-52)

We know, from the definition of capacitance that:

(Eq B-53)

Hence:

(Eq B-54)

Defining the electric field as:

(Eq B-55)

We finally get the result:

(Eq B-56)

Similarly, for rotary electro-static devices:

(Eq B-57)

Which yields a similar result for specific force or electro-static pressure.

B.3.3 *Hydraulic and Pneumatic – Static Conditions*

Under static condiions, the hydraulic/pneumatic actuator pressure is simply the supply pressure available from the pump.

B.3.4 *Summary*

	Magneto-static	Electro-static	Hydraulic (Static)	Pneumatic (Static)
Pressure Equation				
Typical Values (in air):				
Typical Pressures				

The specific force capabilities (force per unit active area) of large scale electrostatic devices in air are several orders of magnitude lower than magnetic devices.

B.4 Micromechanical Systems (MEMs)

With miniature scale devices it is possible to operate at much higher electric fields before breakdown occurs. The effect was first observed by Paschen.

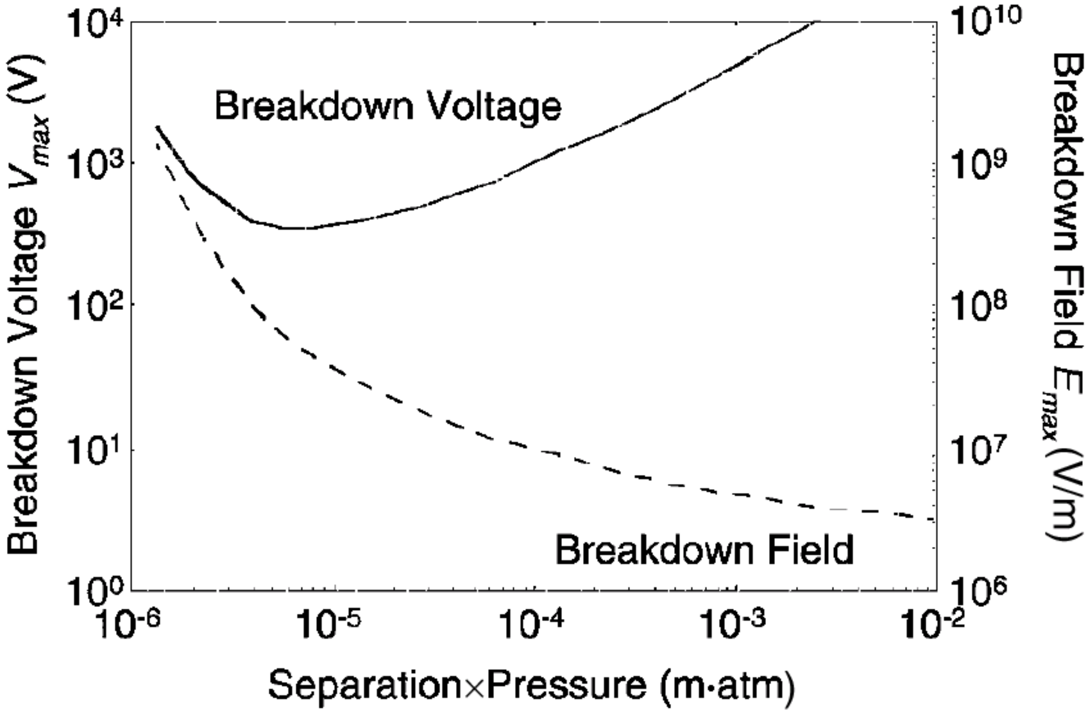
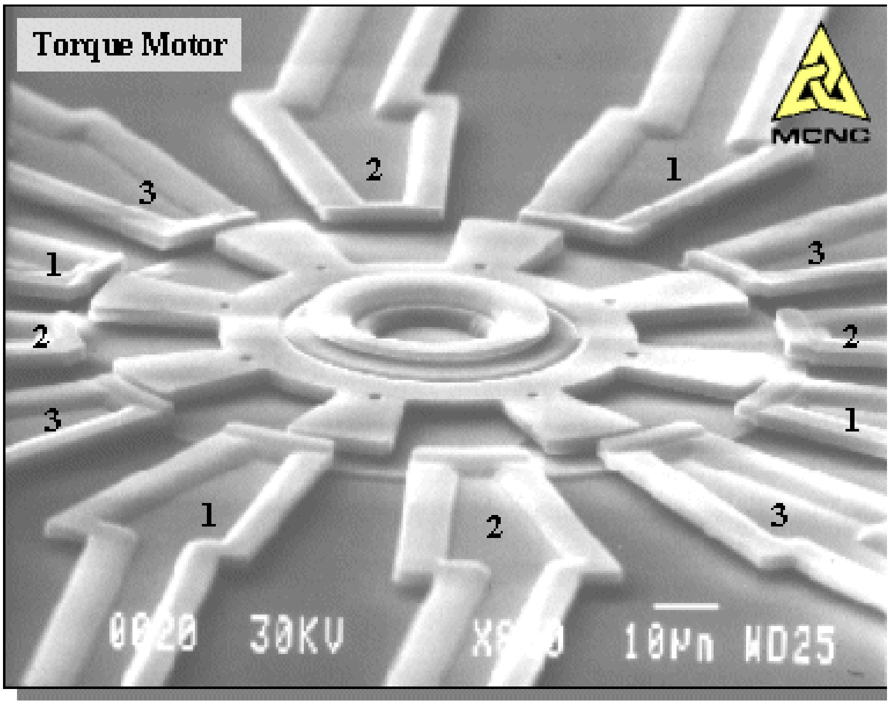


Figure B-7: Paschen Curve

At µm scales the maximum electrical field in air increases by a factor of 100 compared to cm scale devices. Thus the specific force capability with increase by 10^4 and is therefore comparable to magnetic systems. This effect is exploited in micro-mechanical devices, such as the micro-motor shown below.

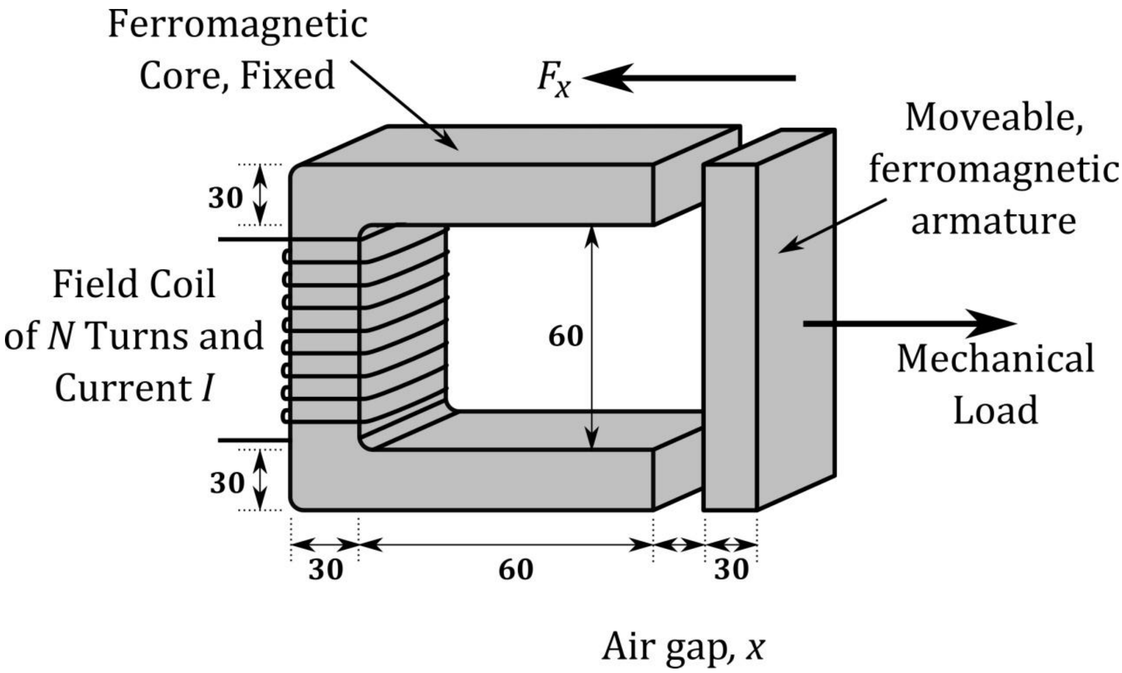


Electrodes are actuated in sequence to produce rotation:

Torque motor uses four sets of three-phase actuators to excite the rotor.

Figure B-8: Micromotor-source MCNC (www.mcnc.org)

EXAMPLE B-1 : LINEAR FERROMAGNETIC ACTUATOR



a) Given the relative permeability μ_r of the core and moveable armature is equal to 1000 and the core does not saturate ,determine an expression for the electromagnet inductance as a function of airgap in the form:

$$L(x) = \frac{a}{2x + b}$$

The coil has 2000 turns. Assume fringing fields across the airgap increase the effective airgap area by 10%.

b) The actuator acts against a constant 10 N spring load. Find:
   i) The current required to overcome the load when the airgap separation is 2mm
   ii) The final position of the actuator if this current is maintained
   iii) The value of current below which the armature will be released

c) Plot a Ψ-I characteristics of the device for a 2mm airgap separation and for the actuator fully closed. Use the Ψ-I diagram to calculate the interchanges of magnetic stored energy, mechanical work and electrical input energy when 0.14A is applied to close the actuator from the initial 2mm separation.

SOLUTION

Using:

And

Where the core reluctance. $\Re_c$, :

The armature reluctance :

and the gap reluctance:

Hence:

b) i) Using:

We know that:

with $x = 2\textbf{mm}$

Re-arranging:

ii)

iii)

c) With an opening of $x = 2\textbf{mm}$ the device inductance:

Therefore the flux linkage corresponding to a) is:

With the actuator fully closed the inductance L increases to

Assuming the current remains constant during the closure of the actuator, $x = 2\text{mm}$ to $x = 0\text{mm}$, the flux linkage with the actuator closed is:

The current changes as follows:

Hence, the $\Psi - I$ diagram for this event (again assuming no saturation) is given below:

The various energies area:

Initial stored magnetic:

Final stored magnetic

Energy transfer from electrical system

Mechanical work

Finally as a check, we know that:

NOTE: If you calculate the work done on the armature:

This is much less that the 0.223 J calculate mechanical energy. In, fact the unaccounted 90% of the energy goes into accelerating the armature. This energy is lost as the closing velocity is high and hence there will be a dissipation of energy (as deformation and therefore heat, sound) when the armature snaps shut.